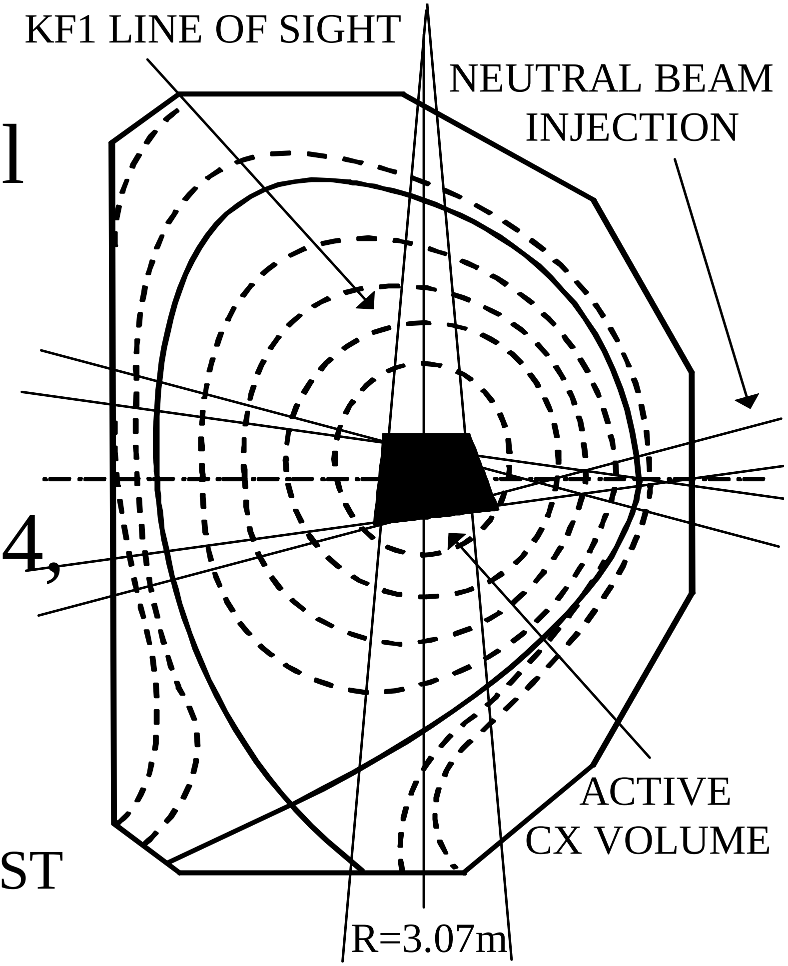
# Error Estimation and Parameter Dependence of the Calculation of the Fast Ion Distribution Function, Temperature and Density Using Data From the KF1 High Energy NPA on JET

Christian Schlatter, Duccio Testa,
Marco Cecconello, Marko Santala,
Andrea Murari

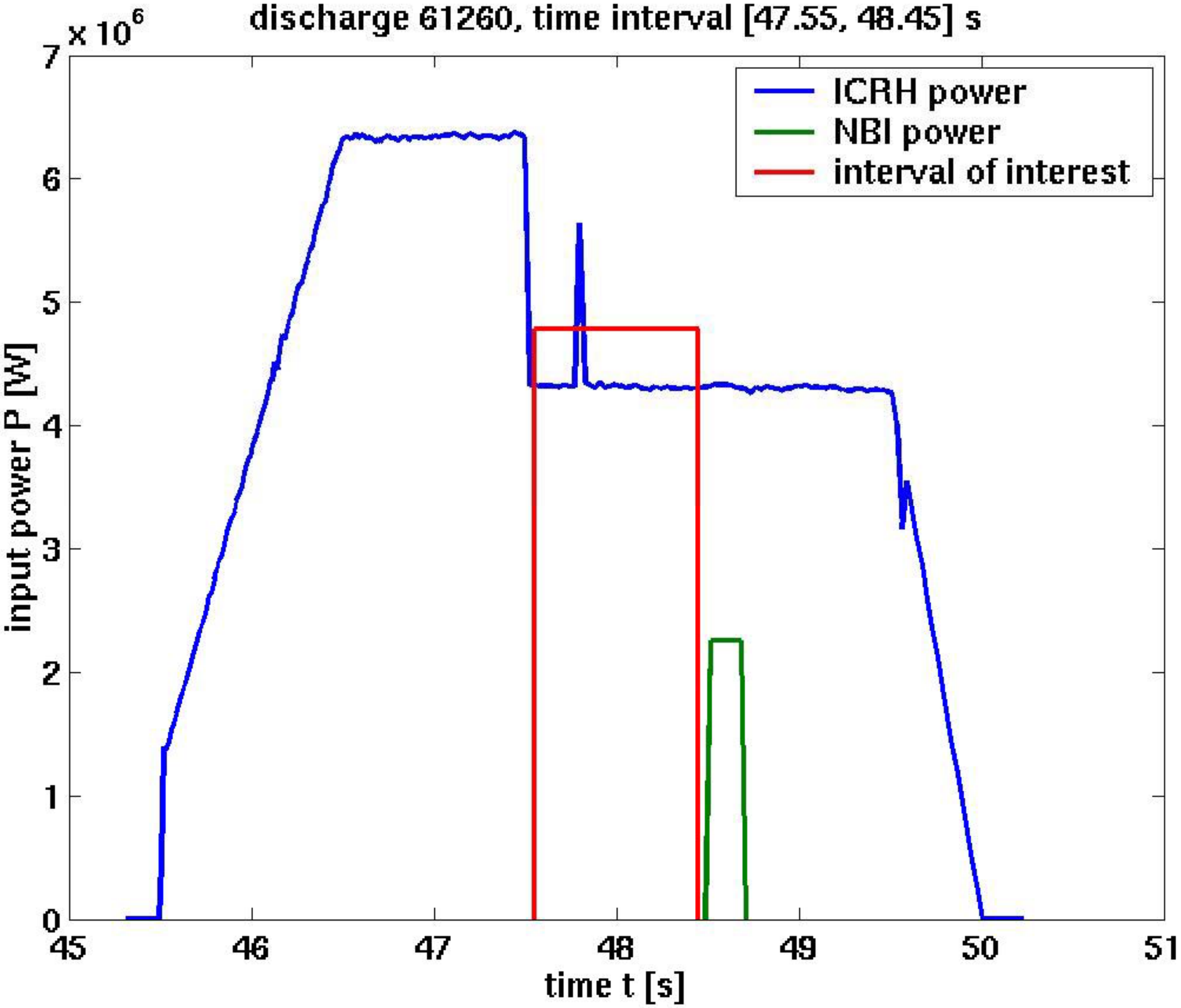
# Introduction : KF 1

- High energy NPA, neutral flux measurement of hydrogen and helium isotopes up to 4 MeV
- Vertical line of sight, oct.4, R = 3.07 m
- Measurement of fast ion $f_{iFAST}(E)$, $T_{iFAST\perp}$ and $n_{iFAST}$

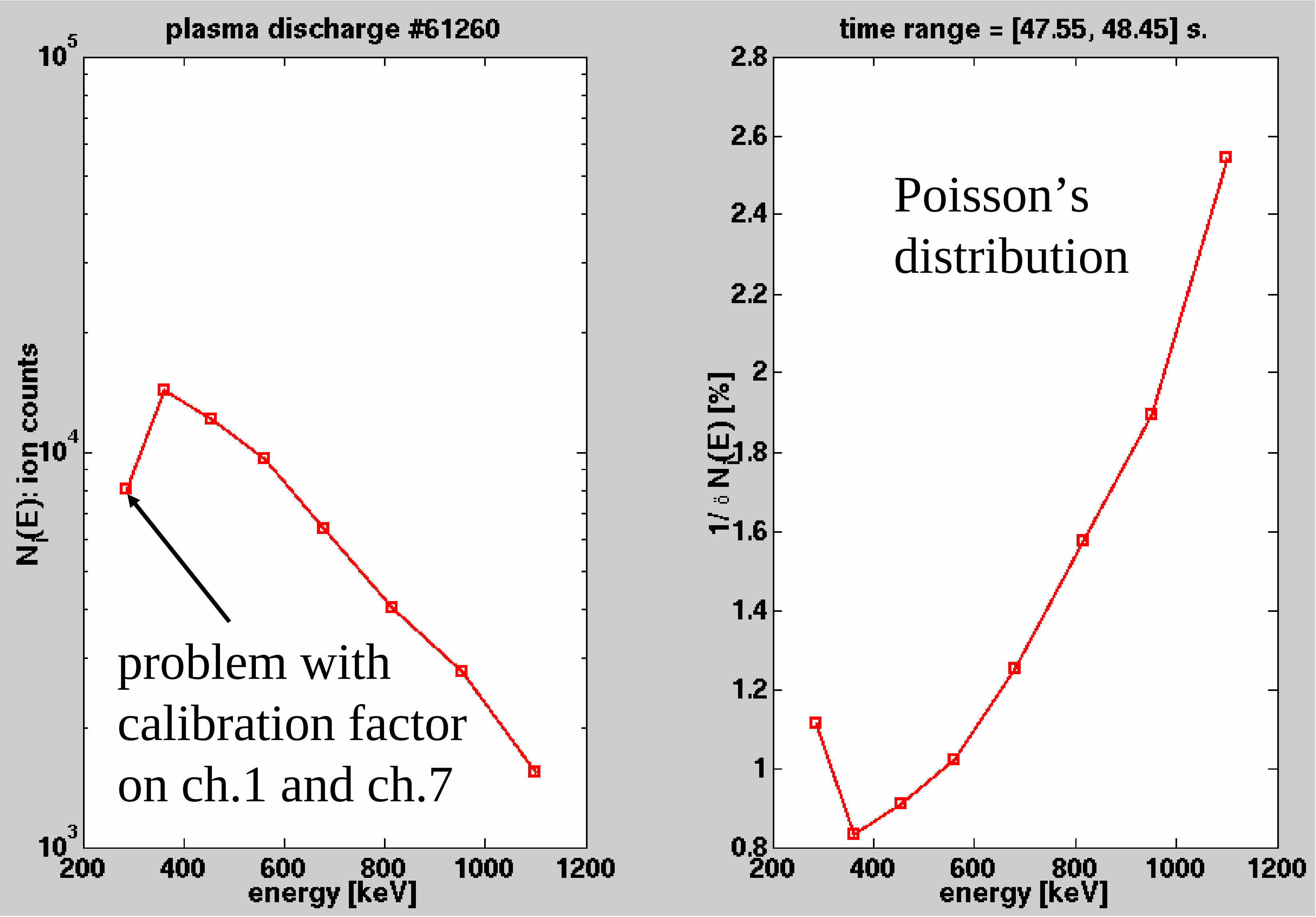


# discharge #61260

- pT-fusion expt
- H-minority heating, 1st harmonic
- $B_T$=3.4 T,
- $I_p$ = 1.8 MA,
- $T_e$ = 7 keV,
- $n_e = 3\cdot10^{-19}$ $m^{-3}$,
- $W_{DIA}$ = 2.5 MJ



# Flux and its error bars



problem with
calibration factor
on ch.1 and ch.7

Poisson’s
distribution

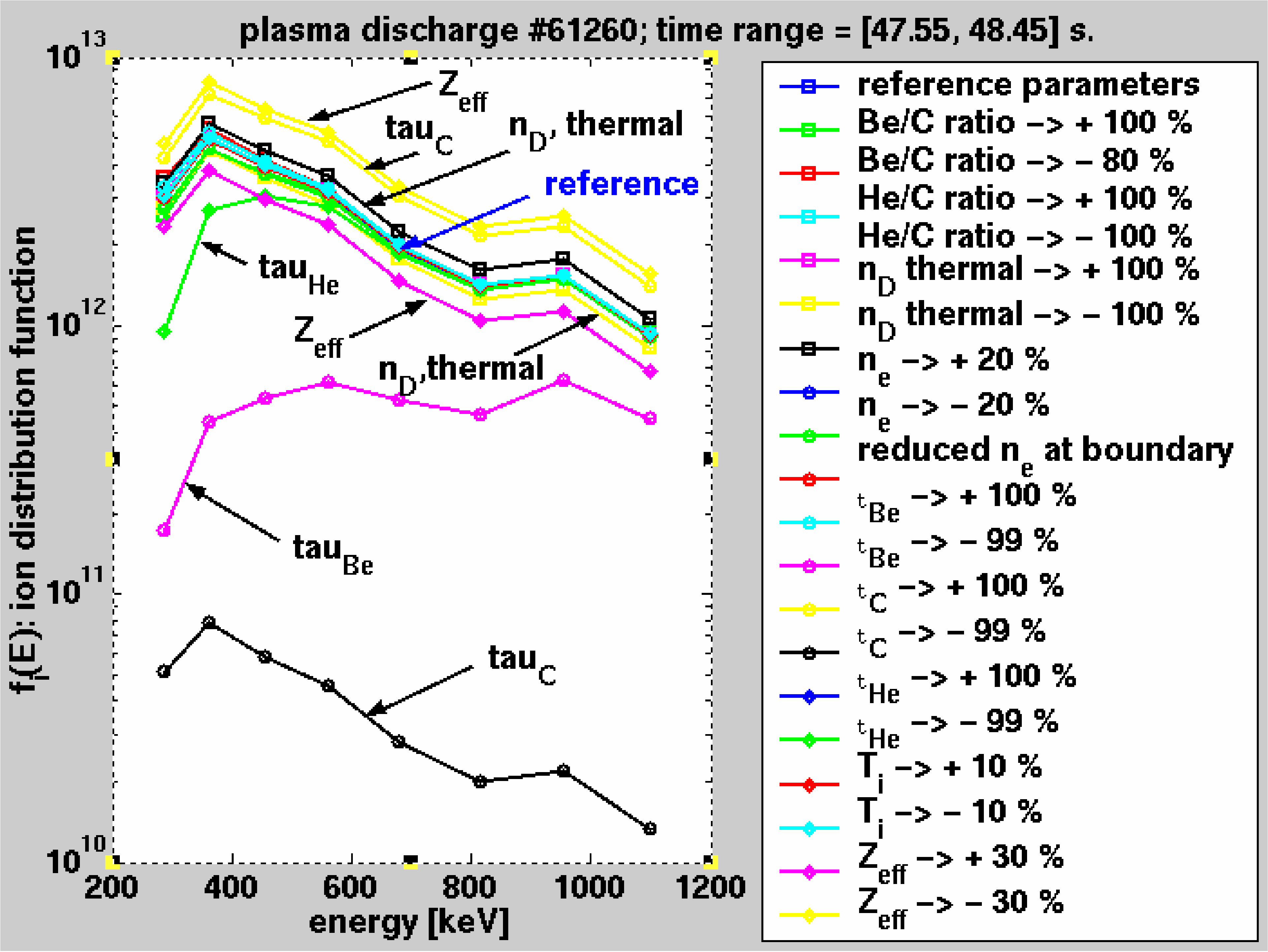
# Fast ion distribution function $f_i(E)$

- $$\mathbf{N(E) = (\Omega S) \cdot \Delta E \cdot \mu(E) \cdot \gamma(E) \cdot P_{\upsilon}(E) \cdot f_i(E)}$$
- $N(E)$: neutral count rate
- $f_i(E)$: fast ion distribution function
- $P_{\upsilon}(E)$: neutralization probability
- $\gamma(E)$: plasma transparency (re-ionization probability)
- $\Delta E$: energy width of the detector
- $\mu(E)$: detection efficiency
- $(\Omega S)$: étendue of the detector

# Neutralization probability $P_\upsilon(E)$

- **I**mpurity **I**nduced **N**eutralization model (IIN)

  A.A.Korotkov et al., NF **37** (1997) 35.
- system of steady-state ion density balance equations for **bare**, **[H]-** and **[He]**-like impurities.
- RR with electrons, CX with impurities, thermal deuterium and NBI atoms.
- $P_\upsilon(E) = \sum_q \langle \sigma \upsilon \rangle_{CX_q/RR_q} \cdot n_q$
- Input parameters: impurity (He, Be, C) density ratios and confinement times, $T_i$, $n_e$, $n_{D,thermal}$, $Z_{eff}$


plasma discharge #61260; time range = [47.55, 48.45] s.
f_i(E): ion distribution function
energy [keV]
10^13
10^12
10^11
10^10
200
400
600
800
1000
1200
Z_eff
tau_C
n_D, thermal
reference
tau_He
Z_eff
n_D,thermal
tau_Be
tau_C
reference parameters
Be/C ratio -> + 100 %
Be/C ratio -> − 80 %
He/C ratio -> + 100 %
He/C ratio -> − 100 %
n_D thermal -> + 100 %
n_D thermal -> − 100 %
n_e -> + 20 %
n_e -> − 20 %
reduced n_e at boundary
t_Be -> + 100 %
t_Be -> − 99 %
t_C -> + 100 %
t_C -> − 99 %
t_He -> + 100 %
t_He -> − 99 %
T_i -> + 10 %
T_i -> − 10 %
Z_eff -> + 30 %
Z_eff -> − 30 %

# Fast ion perpendicular temperature $T_{i\perp}$

- Distribution function of ICRF heated ions (*Stix, NF 15 (1975) 737*)

$$\bar{f}_i(E) \propto \frac{\sqrt{E}}{\bar{T}_\perp}\exp\left(-\frac{E}{\bar{T}_\perp}\right)$$

- inferred temperature

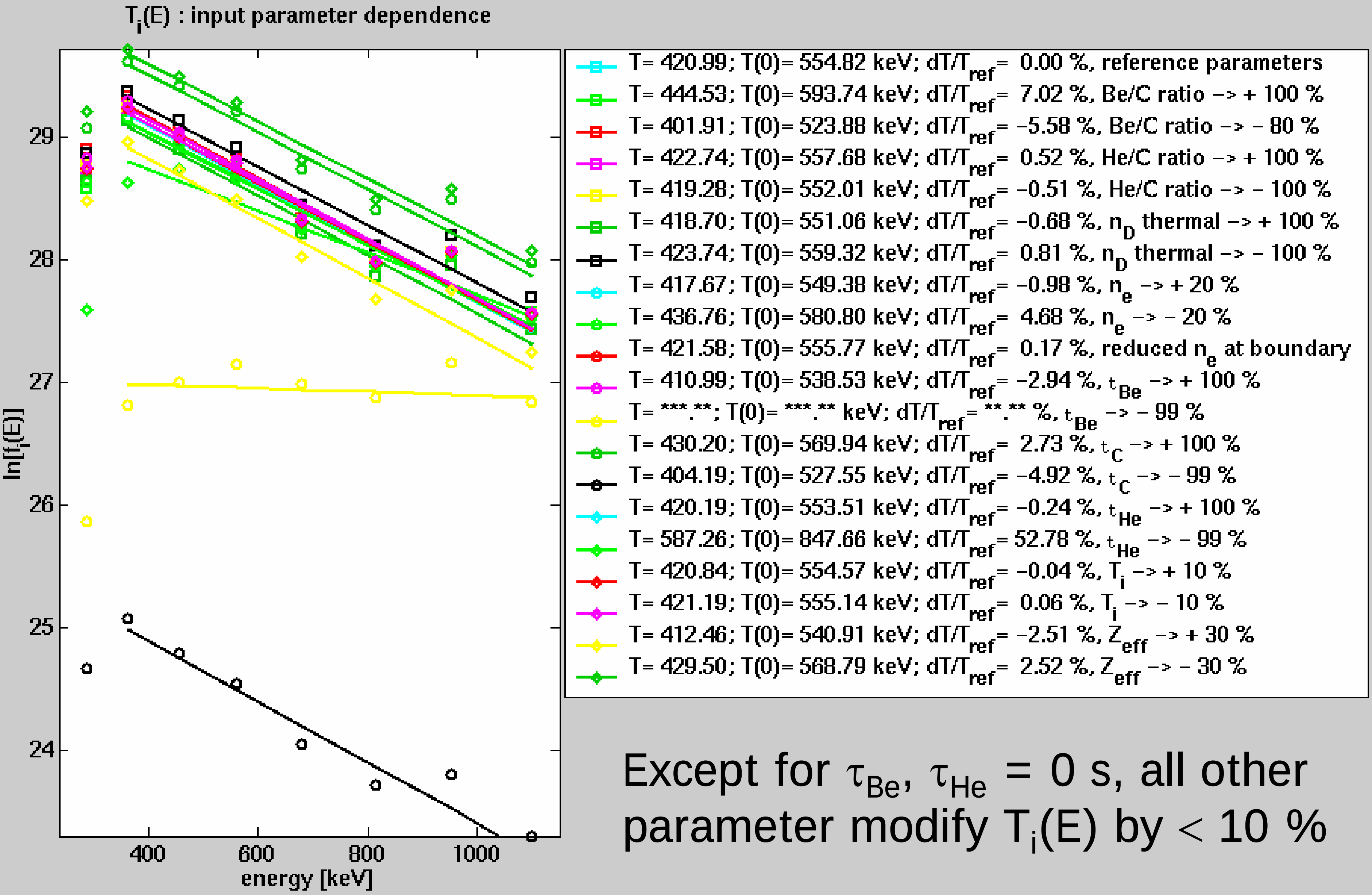
$$\frac{\partial}{\partial E}\ln\frac{\bar{f}_i(E)}{\sqrt{E}} = -\frac{1}{\bar{T}_\perp}$$

- Central perpendicular temperature

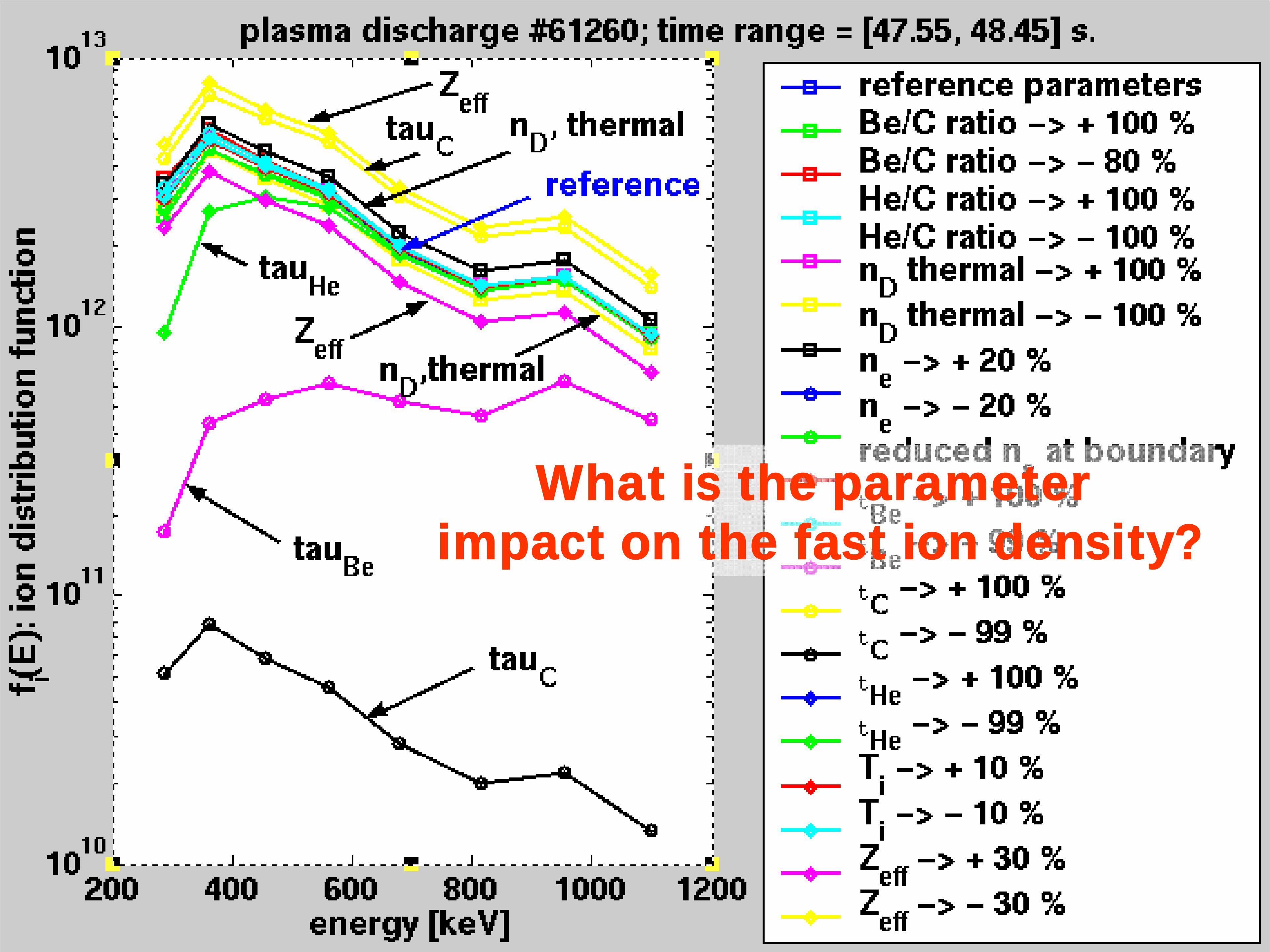
*McClements et al, NF 37 (1997) 4*

$$T_\perp(0) \cong \bar{T}\left(E^*\right)_\perp\left(1+\frac{\bar{T}_\perp\left(E^*\right)}{2E^*}\right)$$

# $T_i$(E,input parameter)



Except for $\tau_{Be}$, $\tau_{He}$ = 0 s, all other parameter modify $T_i$(E) by < 10 %



What is the parameter impact on the fast ion density?

# Fast ion density $n_i$

- NPA:

$$n_i^{fast} = \frac{1}{E_{\max} - E_{\min}} \int_{E_{\min}}^{E_{\max}} f_i(E) \cdot dE$$

- Spectroscopy:

$$\alpha = \frac{H_\alpha}{H_\alpha + D_\alpha + T_\alpha} \propto \frac{n_H}{n_H + n_D + n_T}, \qquad n_D = n_e$$

$$n_i^{fast} = \frac{\alpha}{1-\alpha} n_e$$

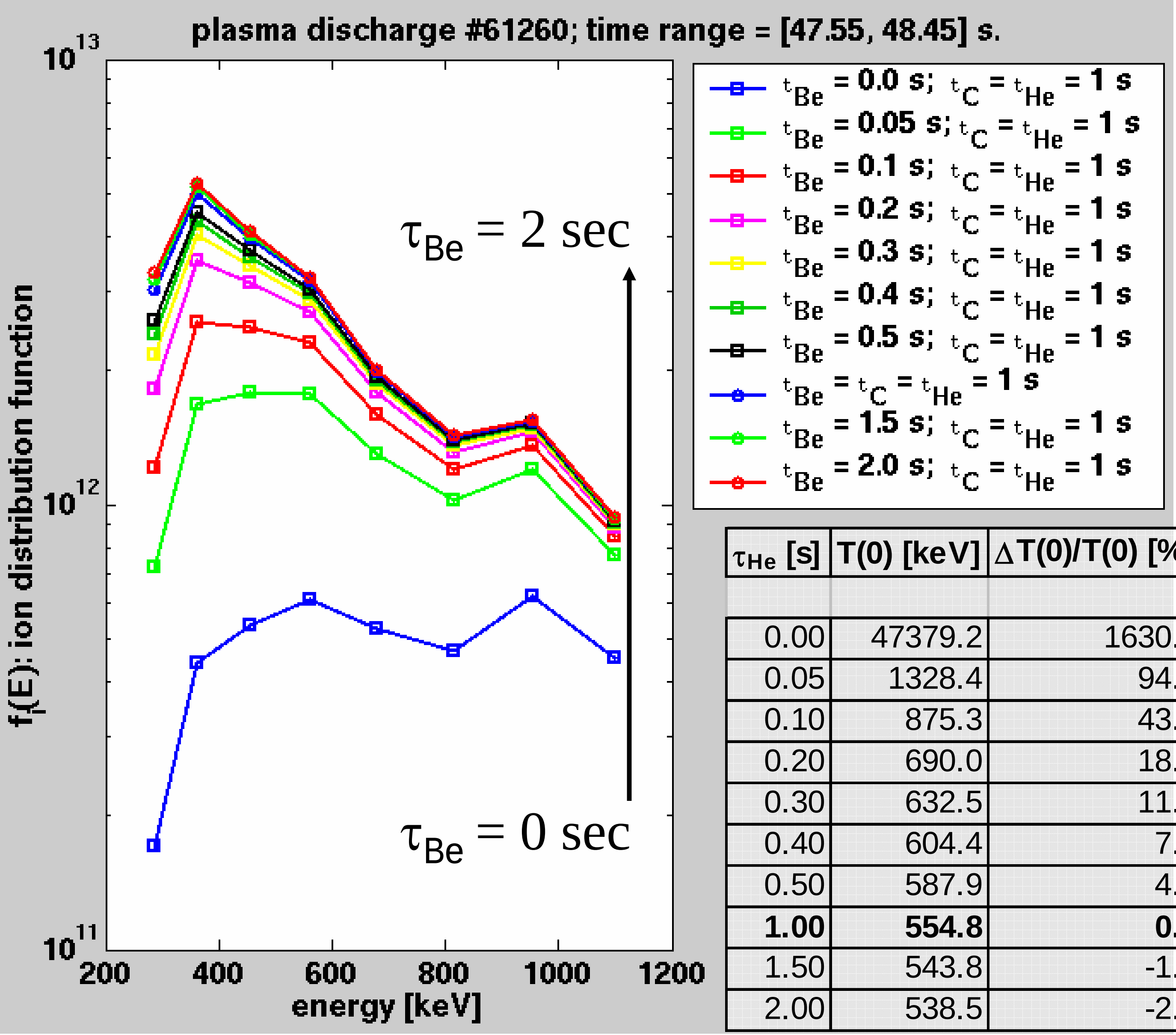
➔ $1.5 \cdot 10^{18}$ m$^{-3}$

- Fast particle energy measurement (NF33(1993)7)

$$W_{fast} = 4\pi^2 R_0 \int_0^a r \cdot \kappa(r) \cdot n_i^{fast}(r) \cdot \left[ T_\perp + \frac{1}{2} T_\parallel \right] \cdot dr = \frac{4}{3}\left( W_{DIA} - W_{MHD} \right)$$

Gaussian density and temperature profiles: ➔ $1.2 \cdot 10^{18}$ m$^{-3}$

# Scan of $\tau_{Be}$



$$\Gamma_Z = -Dn_Z' + \frac{r}{a}Vn_Z$$

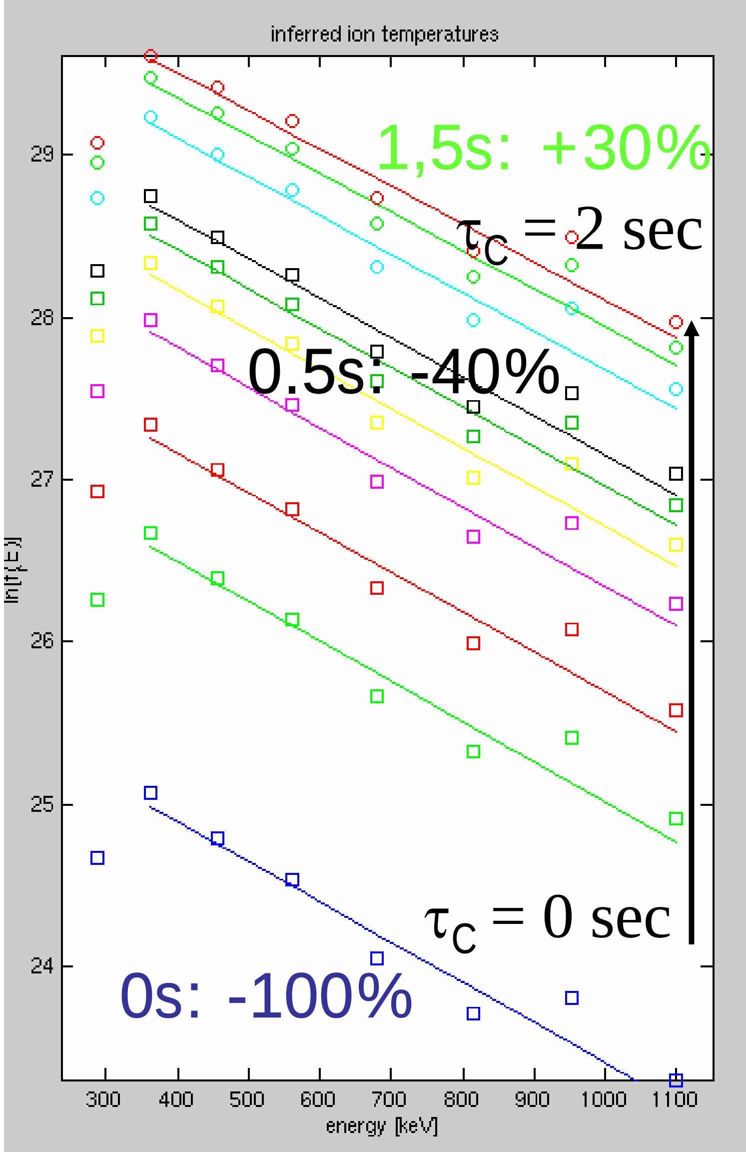
$$\tau_Z = \frac{n_Z}{div\Gamma_Z}$$

$$\Rightarrow \bar{\tau}_Z \approx 1s$$

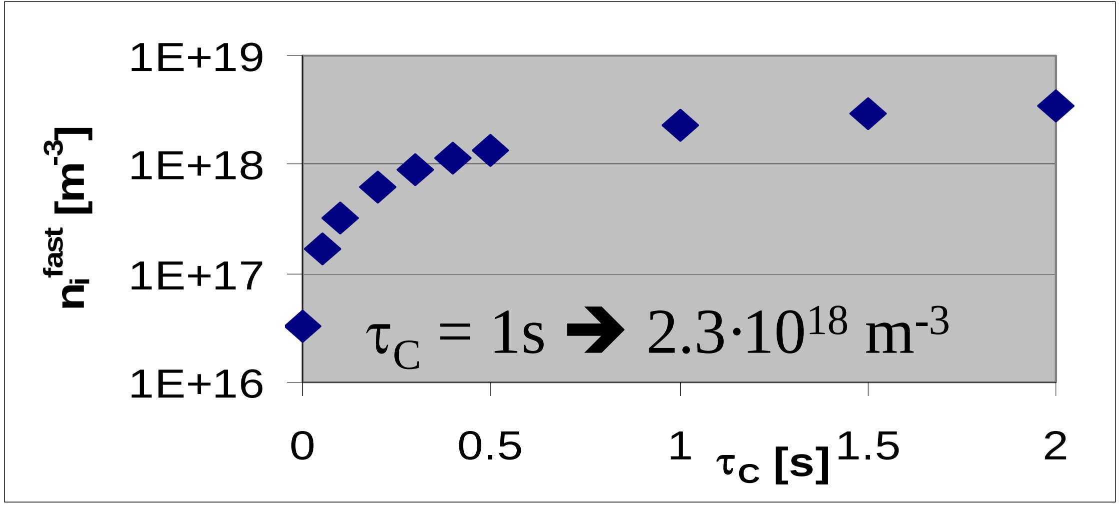
*NF* 37(1997)35

| $\tau_{He}$ [s] | T(0) [keV] | $\Delta$T(0)/T(0) [%] | $n_i^{fast}$ [$m^{-3}$] | $\Delta n_i^{fast}/n_i^{fast}$ [%] |
|---|---|---|---|---|
| | | | | |
| 0.00 | 47379.2 | 1630.9 | 5.3E+17 | -76.8 |
| 0.05 | 1328.4 | 94.9 | 1.3E+18 | -42.2 |
| 0.10 | 875.3 | 43.0 | 1.7E+18 | -26.9 |
| 0.20 | 690.0 | 18.9 | 2.0E+18 | -14.6 |
| 0.30 | 632.5 | 11.0 | 2.1E+18 | -9.2 |
| 0.40 | 604.4 | 7.1 | 2.2E+18 | -6.2 |
| 0.50 | 587.9 | 4.8 | 2.2E+18 | -4.3 |
| **1.00** | **554.8** | **0.0** | **2.3E+18** | **0.0** |
| 1.50 | 543.8 | -1.6 | 2.3E+18 | 1.6 |
| 2.00 | 538.5 | -2.4 | 2.3E+18 | 2.3 |

# Impact on $n_i$: scan of $\tau_C$



🟦 T= 404.19; T(0)= 527.55 keV; $dT/T_{ref}$= −4.92 %, $t_C$ = 0.0 s; $t_{Be}$ = $t_{He}$ = 1 s
🟩 T= 404.92; T(0)= 528.72 keV; $dT/T_{ref}$= −4.70 %, $t_C$ = 0.05 s; $t_{Be}$ = $t_{He}$ = 1 s
🟥 T= 406.10; T(0)= 530.62 keV; $dT/T_{ref}$= −4.36 %, $t_C$ = 0.1 s; $t_{Be}$ = $t_{He}$ = 1 s
🟪 T= 408.22; T(0)= 534.05 keV; $dT/T_{ref}$= −3.74 %, $t_C$ = 0.2 s; $t_{Be}$ = $t_{He}$ = 1 s
🟨 T= 410.23; T(0)= 537.30 keV; $dT/T_{ref}$= −3.16 %, $t_C$ = 0.3 s; $t_{Be}$ = $t_{He}$ = 1 s
🟩 T= 412.15; T(0)= 540.40 keV; $dT/T_{ref}$= −2.60 %, $t_C$ = 0.4 s; $t_{Be}$ = $t_{He}$ = 1 s
⬛ T= 413.85; T(0)= 543.17 keV; $dT/T_{ref}$= −2.10 %, $t_C$ = 0.5 s; $t_{Be}$ = $t_{He}$ = 1 s
🟦 T= 420.99; T(0)= 554.82 keV; $dT/T_{ref}$= 0.00 %, $t_C$ = $t_{Be}$ = $t_{He}$ = 1 s
🟩 T= 426.33; T(0)= 563.57 keV; $dT/T_{ref}$= 1.58 %, $t_C$ = 1.5 s; $t_{Be}$ = $t_{He}$ = 1 s
🟥 T= 430.20; T(0)= 569.94 keV; $dT/T_{ref}$= 2.73 %, $t_C$ = 2.0 s; $t_{Be}$ = $t_{He}$ = 1 s



α–line measurements: $1.5\cdot10^{18}$ m$^{-3}$ ± 30%

Magn. measurements: $1.2\cdot10^{18}$ m$^{-3}$ ± 30%

# What about the error bars?

- $$\frac{\Delta f}{f} \approx \sqrt{\left(\frac{\Delta P_\upsilon}{P_\upsilon}\right)^2 + \left(\frac{\Delta \gamma}{\gamma}\right)^2 + \left(\frac{\Delta \mu}{\mu}\right)^2 + \left(\frac{\Delta N}{N}\right)^2} =$$
  $$\sqrt{0.45^2 + 0.15^2 + 0.1^2 + 0.05^2} \approx 50\% \approx \frac{\Delta n}{n}$$
- The main source of uncertainties is the [H] electron donor density in the plasma core
- This can be improved by a better analysis of the input parameters
- $$\frac{\Delta T_\perp}{T_\perp} \approx 2 \cdot 10^{-3} \frac{\Delta \sigma_{CX}}{\sigma_{CX}} T_\perp \approx 10\%$$
- Main source: calculated $C^5$-ions CX-cross-section (20 %)
- NF 37 (1997) 35, NF 40 (2000) 975

# Conclusion

- Very reliable and robust perpendicular fast core ion temperature measurement
- Typical “crude IIN model” fast particle density measurement has ~50% uncertainty
  - refined analysis can bring it down
  - measurement consistent with edge spectroscopy and fast ion energy from magnetics
- **Detectors need recalibration**